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# Two Exterior Griffth Cracks Opened By Forces At The Crack Faces.

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Abstract

In the study of physical quantities the structure face the problem of crack's presence. We use Integral equation to solve such problems. In this paper we deals with two exterior Griffth cracks which are opened by force at crack surface. Here we get the integral equations which are reduced to Fredholm integral Equatoins, It is easy to make exterior Griftth crack to perform the experiment. Hence the problem is of physical importance and special case of loading is discussed.

The cracks occupy the regions as y = 0,

$b \leq |x| < \infty$ the common Interface and mathematically the problem is reduced to the following boundary value problem, see fig 5.1 and 5.2

$$\sigma_{xy}(x,0^+) = \sigma_{xy}(x,0^-) = 0\,,\ b \leq |x| < \infty \quad (5.1.1)$$

$$\left.\begin{aligned} \sigma_{yy}\left(x,0^+\right) &= -P_1(x) \\ \sigma_{yy}\left(x,0^-\right) &= -P_2(x) \end{aligned}\right\} \quad b \leq |x| < \infty \qquad (5.1.2)$$

and the following continuity conditions.

$$\sigma_{xy}(x,0^+) = \sigma_{xy}(x,0^-) \qquad 0 \leq |x| < b \qquad (5.1.3)$$

$$\sigma_{yy}(x,0^+) = \sigma_{yy}(x,0^-) \qquad (5.1.4)$$

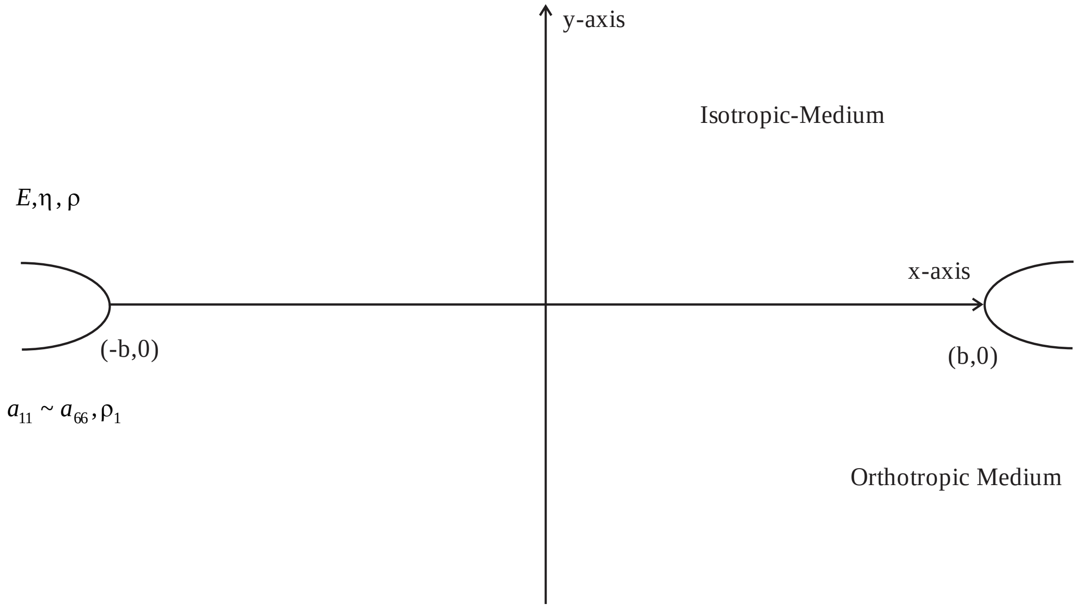


**Fig 5.1 Geometry of problem with Two Exterior Griffith Cracks.**

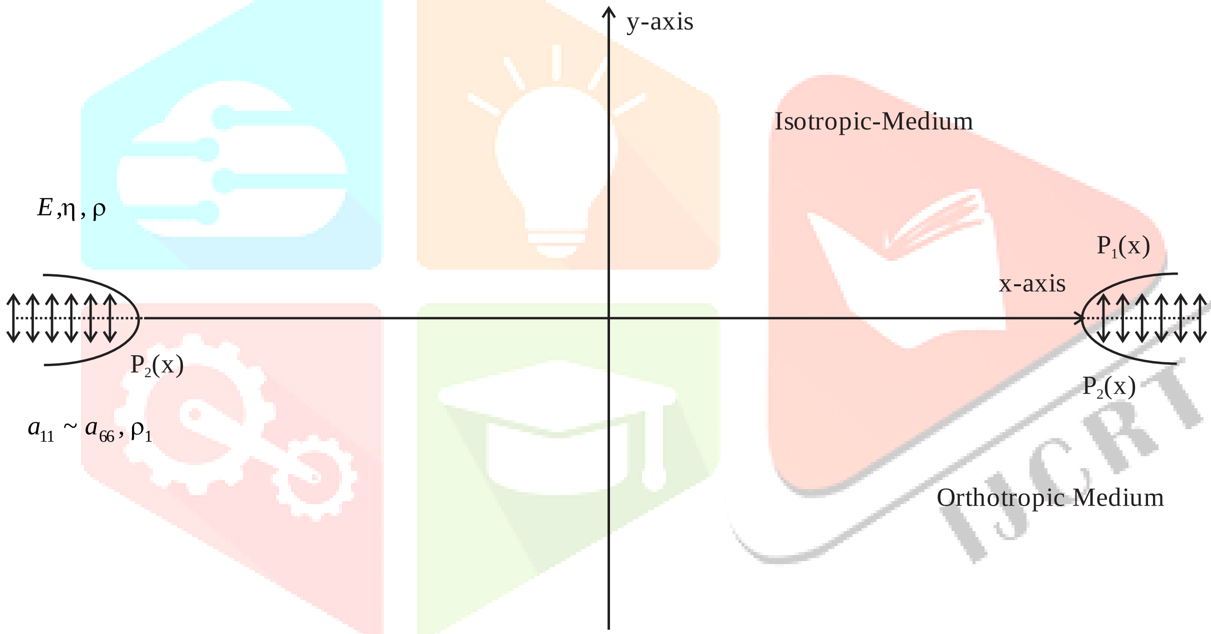


**Fig. 5.2 : Geometry with Boundary and Continuity Conditions.**

$$u_y(x, 0^+) = u_y(x, 0^-) \qquad (5.1.5)$$

$$u_x(x, 0^+) = u_x(x, 0^-) \qquad (5.1.6)$$

We use the symmetry of the geometry of the problem. Thus the above boundary conditions reduce to.

$$\sigma_{xy}(x, 0^+) = \sigma_{xy}(x, 0^-) = 0 \quad b \leq x < \infty \quad (5.1.7)$$

$$\left.\begin{aligned} \sigma_{yy}\left(x, 0^+\right) &= -P_1(x) \\ \sigma_{yy}\left(x, 0^-\right) &= -P_2(x) \end{aligned}\right\} \quad b \leq |x| < \infty \qquad (5.1.8)$$

And

$$\sigma_{xy}(x,0^+) = \sigma_{xy}(x,0^-) \qquad 0 \leq x < b \qquad (5.1.9)$$

$$\sigma_{yy}(x,0^+) = \sigma_{yy}(x,0^-) \qquad 0 \leq x < b \qquad (5.1.10)$$

$$u_y(x,0^+) = u_y(x,0^-) \qquad 0 \leq x < b \qquad (5.1.11)$$

$$u_x(x,0^+) = u_x(x,0^-) \qquad 0 \leq x < b \qquad (5.1.12)$$

Thus we have to solve the equations of equilibrium

$$\left.\begin{aligned} \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xx}}{\partial y} = 0 \\ \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0 \end{aligned}\right\} \qquad (5.1.13)$$

Along with stress-strain relations

(i) Isotropic case

$$\left.\begin{aligned} 2\mu e_{xx} = (1-\eta)\sigma_{xx} - \eta\sigma_{xy} \\ 2\mu e_{yx} = (1-\eta)\sigma_{yy} - \eta\sigma_{xx} \end{aligned}\right\}$$

$$2\mu e_{xy} = \sigma_{xy}$$

where $\mu$ and $\eta$ are modulus of rigidity and poisson's ratio of isotropic medium, $e_{xx}$, $e_{yy}$, $e_{xy}$ are elastic strains.

(ii) Orthotropic case

$$\left.\begin{aligned}\frac{\partial u_x}{\partial x} &= a_{11}\sigma_{xx} + a_{12}\sigma_{yy} \\ \frac{\partial u_y}{\partial y} &= a_{12}\sigma_{xx} + a_{22}\sigma_{yy} \\ \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} &= a_{66}\sigma_{xy}\end{aligned}\right\}$$

The plan of the chapter is as follows. In section 5.2 the problem is reduced to coupled dual integral equations. The solutions of these dual integral equations are then reduced to coupled Fredholm integral equations of second kind in sections 5.3. The solution of Fredholm integral equations will be given in section 5.4. Section 5.5 will present the physical quantities in terms of solution of these Fredholm integral equations. Last section 5.6 will deal with one special case of force at crack surface.

**5.2 REDUCTION TO DUAL INTEGRAL EQUATIONS.**

To solve the above mentioned problem, we assume the solution of equations of equilibrium (5.1.13) and the stress strain relations (i), (ii) for y>0 and y<0 cases respectively as.

$$u_y(x,y) = \frac{2(1+\eta)}{E}\int_0^\infty \frac{\cos(\xi x)}{x^2}$$

$$\left[(1-\eta)\frac{\partial^3}{\partial y^3}H(\xi,y) + (\eta-2)\xi^2\frac{\partial}{\partial y}H(\xi,y)\right]d\xi \qquad (5.2.1)$$

$$u_y(x,y) = \frac{2(1+\eta)}{E}\int_0^\infty \frac{\sin(\xi x)}{\xi}$$

$$\left[(1-\eta)\frac{\partial^2}{\partial y^2}H(\xi,x) + \eta\xi^2 H(\xi,y)\right]d\xi \qquad (5.2.2)$$

With

$$H(\xi,x) = \left[A(\xi) + yB(\xi)\right]e^{-\xi y} \qquad (5.2.3)$$

$$u_y(x,y)=\int_0^\infty \frac{\cos(\xi x)}{\xi^2}\left[a_{11}G,_{yyy}-\xi^2\left(a_{12}+a_{66}\right)G,y\right]d\xi \quad (5.2.4)$$

$$u_x(x,y)=\int_0^\infty \frac{\cos(\xi x)}{\xi^2}\left[a_{11}G,_{yyy}-\xi^2\left(a_{12}+a_{66}\right)G,y\right]d\xi \quad (5.2.4)$$

$$u_x(x,y)=\int_0^\infty \frac{\sin(\xi x)}{\xi^2}\left[a_{11}G,_{yy}-a_{12}\xi^2 G\right]d\xi \quad (5.2.5)$$

With

$$G(\xi,y)=\frac{1}{r_1-r_2}\left[\left\langle (r_1-r_2)C(\xi)D(\xi)\right\rangle e^{r_1\xi_y}+D(\xi)e^{r_2\xi_y}\right]d\xi \quad (5.2.6)$$

Where $C(\xi)$ $C(\xi)$ and $D(\xi)$ are two arbitrary constants and $r_1$ and $r_2$ two roots of $r^4+B_1r^2+B^2=0$

Where

$$\left.\begin{aligned} B_1 &= \frac{2(a_{12}+a_{66})}{a_{11}} \\ B_2 &= \frac{a_{22}}{a_{11}} \end{aligned}\right\}$$

The boundary conditions (5.1.7)-(5.1.8) along with continuity conditions (5.1.9)-(5.1.10) and reducing them for x > 0 only, we get, after Fourier inversion as.

$$\xi\left[r_1C(\xi)-D(\xi)\right]+B(\xi)-\xi A(\xi)=0 \quad (5.2.7)$$

$$A(\xi)-B(\xi)=\overline{p_1}(\xi)/\xi^2 \quad (5.2.8)$$

With

$$\overline{p_1}(\xi)=\frac{2}{\pi}\int_0^\infty\left[p_1(x)-p_2(x)\right]\cos(\xi x)dx \quad (5.2.9)$$

Thus making use of (5.2.7)-(5.2.8) and the continuity conditions for displacement components (5.1.11)-(5.1.12) and the relations.

$$\sigma_{xy}(x,0^+) + \sigma_{xy}(x,0^-) = 0 \qquad (5.2.10)$$

$$\sigma_{yy}(x,0^+) + \sigma_{yy}(x,0^-) = -[P_1(x) + P_2(x)] \qquad (5.2.11)$$

We get

$$\int_0^\infty \xi \sin(\xi x)\left[K_5 C(\xi) + K_6 D(\xi)\right] d\xi = P_4(x), \quad 0 \le x \le b \qquad (5.2.12)$$

$$\int_0^\infty \xi \cos(\xi x)\left[K_7 C(\xi) + K_8 D(\xi)\right] d\xi = P_5(x), \quad 0 \le x \le b \qquad (5.2.13)$$

$$\int_0^\infty \xi^2 \sin(\xi x) C(\xi)\, d\xi = P_6(x). \qquad b < x < \infty \qquad (5.2.14)$$

$$\int_0^\infty \xi^2 \cos(\xi x) C(\xi)\, d\xi = P_7(x). \qquad b < x < \infty \qquad (5.2.15)$$

With

$$\left.\begin{aligned} P_4(x) &= \int_0^\infty \frac{P_1(\xi)\sin(\xi x)\, d\xi}{\xi} \\ P_5(x) &= \int_0^\infty \frac{P_1(\xi)\cos(\xi x)\, d\xi}{\xi} \end{aligned}\right\} \qquad (5.2.16)$$

$$\left.\begin{aligned} P_6(x) &= \frac{1}{1+r_1}\int_0^\infty P_1(\xi)\sin(\xi x)\, d\xi \\ P_6(x) &= \frac{1}{2}\left[P_1(x) + P_1(x)\right] - \frac{1}{2}\int_0^\infty P_1(\xi)\cos(\xi x)\, d\xi \end{aligned}\right\} \qquad (5.2.17)$$

Where K is constants.

**5.3 REDUCTION TO FREDHOLM INTEGRAL EQUATION.**

We assume that

$$\left.\begin{aligned}\xi\left[K_5C(\xi)+K_6D(\xi)\right]=\phi(\xi)\\ \xi\left[K_5C(\xi)+K_6D(\xi)\right]=\psi(\xi)\end{aligned}\right\} \qquad (5.3.1)$$

Where $\phi(\xi)$ and $\psi(\xi)$ are two new functions. Substituting

the values of

$C(\xi)$ **and** $D(\xi)$ from (5.3.1) into (5.2.12) - (5.2.13) we get the triple integral equations.

$$\int_0^\infty \phi(\xi)\sin(\xi x)\,d\xi = P_4(x) \quad 0\le x\le b \qquad (5.3.2)$$

$$\int_0^\infty \phi(\xi)\cos(\xi x)\,d\xi = P_5(x) \quad 0\le x\le b \qquad (5.3.3)$$

And

$$\int_0^\infty \xi\left[K_{10}\phi(\xi)-K_{11}\psi(\xi)\right]\sin(\xi x)\,d\xi = P_6(x) \quad b<x<\infty \qquad (5.3.4)$$

$$\int_0^\infty \xi\left[K_{12}\phi(\xi)-K_{13}\psi(\xi)\right]\cos(\xi x)\,d\xi = P_7(x) \quad b<x<\infty \qquad (5.3.4)$$

With

$$\left.\begin{aligned} K_{10} = K_8 / K_9, K_{11} = K_7 / K_9 \\ K_{12} = r_1 K_{10} + K_{10} + K_{11} \\ K_{13} = r_1 K_{11} + K_5 / K_9 \\ K_9 = K_5 K_8 - K_6 K_7 \end{aligned}\right\} \qquad (5.3.6)$$

To solve above triple equations we use the method of Srivastava and Lowengrub [79], we assume.

$$\pi\xi\varphi(\xi) = 2\left[\int_b^\infty g(t)\langle 1-\cos\xi t\rangle dt - \int_0^b P_4(t)\langle 1-\cos\xi t\rangle dt\right] \qquad (5.3.7)$$

$$\pi\xi\varphi(\xi) = 2\left[\int_b^\infty h(t)\sin\xi t dt - \int_0^b P_5(t)\sin\xi t dt\right] \qquad (5.3.8)$$

Then (5.3.2) - (5.3.3) will be satisfied identically if

$$\left.\begin{aligned} \int_b^\infty g(t)dt = P_4(b) \\ \int_b^\infty h(t)dt = P_5(b) \end{aligned}\right\} \qquad (5.3.9)$$

Now the substitution of (5,3,7) – (5.3.8) into

(5.3.4) -(5.3.5) and then evaluating the integral and then using the method of Srivastava and lowengrub [79] for inversion. We get for

$b < x < \infty$

$$g(t) = \frac{1}{\pi K_{11}\delta(t)}\left[\int_b^\infty \frac{x\delta(x)P_{10}(x)dx}{x^2 - t^2} + D_1\right] \qquad (5.3.10)$$

$$h(t) = \frac{1}{\pi K_{11}\delta(t)}\left[\int_b^\infty \frac{x\delta(x)P_{11}(x)dx}{x^2 - t^2} + D_2\right] \qquad (5.3.11)$$

Where

$$P_{10}(x) = P_9(x) - K_{13}h(x) \tag{5.3.12}$$

$$P_{11}(x) = P_8(x) + K_{10}g(x) \tag{5.3.13}$$

$$P_8(x) = P_7(x) - \frac{4K_{11}}{\pi}\int_0^b \frac{tP_5(t)dt}{t^2 - x^2} \tag{5.3.14}$$

$$P_9(x) = P_6(x) + \frac{4K_{12}}{\pi}\int_0^b \frac{P_4(t)dt}{t^2 - x^2} \tag{5.3.15}$$

$$\delta(t) = \left\{\left(t^2 - b^2\right)\right\}^{1/2} \tag{5.3.16}$$

And $D_1$ and $D_2$ being two arbitrary constants. These will be evaluated through (5.3.10) - (5.3.11) and (5.3.9). The relations (5.3.10) - (5.3.11) are coupled Fredholm integral equations of second kind. The solution of these equation will be given in next section.

## 5.4 SOLUTION OF FREDHOLM INTEGRAL EQUATIONS.

First, we shall make the coupled Fredhom integral equations decoupled. First we substitute for $P_{11}$ from (5.3.13) into (5.3.11) and then, after interchanging the order of integrations, evaluating the integrals, we get

$$h(t) = P_{12}(t) - \int_b^\infty h(\alpha)K_1(\alpha,t)d\alpha \tag{5.4.1}$$

Where

$$K_1(\alpha,t) = \frac{K_{10}K_{13}}{\pi K_{13}\sqrt{t^2-b^2}}\frac{\alpha\sqrt{\alpha^2-b^2}}{\alpha^2-t^2}\phi_1(t,\alpha) \tag{5.4.2}$$

$$0(t,\alpha) = \frac{1}{2x}\log\left|\frac{b+x}{b-x}\right| \tag{5.4.3}$$

$$P_{12}(t) = \frac{1}{\pi K_{11}\sqrt{t^2 -}}$$

$$+\frac{1}{K_{13}}\int_b^\infty P_9(\alpha)K_1(t,\alpha)d\alpha\Bigg] \qquad b \le t \le \infty \qquad (5.4.4)$$

Thus we get a Fredholm integral equation of second kind in h(x) in (5.4.1) This value of h(t), after getting the solution of Fredholm integration will be substituted to get the solution for g(t). We assume the expansion of b(t) as.

$$b(t) = \sum_{m=0}^{\infty} \delta^m h_m(t)$$

(5.4.5)

Where

$$\delta = K_8 / K_7$$

The numerical value of $\delta$ are in [0,1]. But $\delta = 0$ when the orthotropic medium becomes as isotropic. The value $\delta$ depends upon elastic properties of the medium, Now substituting the value of h(t) from (5.4.5) into (5.4.1) and then equation the coefficients of $\delta^m$ from both sides of the equation, we get.

$h_0(t) = P_{12}(t)$ (5.4.6)

$h_1(t) = -K_{14}I_0(t)$ (5.4.7)

$h_2(t) = -K_{14}[r_0 I_0(t) - K_{14}I_1(t)]$ (5.4.8)

$h_3(t) = K_{14}[-r_0 I_0(t) + r_1(1 + K_{14})I_1(t) - K_{14}I_1(t)]$

(5.4.9)

etc. where

$$I_n(t) = \int_b^\infty K_1(\alpha,t)I_{n-1}(\alpha)\,d\alpha, \qquad n \ge 1 \quad (5.4.10)$$

$$I_n(t) = \int_b^\infty K_1(\alpha,t)P_{12}(\alpha)\,d\alpha, \qquad (5.4.11)$$

With

$$K_{14} = \frac{K_{10}}{K_{11}K_{12}}$$

Thus h(t) will be obtained through (5.4.3)-(5.4.12) . The substitutions of this h(t) into (5.3.10) with gives g(t). In next section we shall be evaluating the physical quantities in the neighbourhood of crack tips.

### 5.5 PHYSICAL QUANTITIES

The physical quantities which are of practical importance, are crack opening displacement components and the stress components in the viscinity of crack tips.

Displacement Components

The values of integrals (5.3.2) - (5.3.3) for

$b \leq x < \infty$ will give us

iso orth

$$u_x(x,0) - u_x(x,0) = P_4(x) - P_4(x) - \int_0^x g(t)dt. \quad b \leq x < \infty \quad (5.5.1)$$

and

Iso orth

$$u_y(x,0) - u_y(x,0) = \int_x^\infty h(t)dt - P_5(x). \quad b \leq x < \infty \quad (5.5.2)$$

Now we evaluate the values of displacement components as.

Iso orth.

$$u_x(x,0) + u_x(x,0) = \int_0^\infty \sin \xi(x) \left[ K_a \phi(\xi) - K_b \Psi(\xi) \right] d\xi.$$

$$+K_0\int_0^\infty \frac{p_1(\xi)\sin\xi x d\xi}{\xi} \qquad b\le x<\infty \quad (5.5.3)$$

Iso orth.

$$u_y(x,0)+u_y(x,0)=\int_0^\infty \cos\xi(x)\left[K_c\phi(\xi)-K_d\Psi(\xi)\right]d\xi.$$

$$+K_0\int_0^\infty \frac{p_1(\xi)\cos\xi x d\xi}{\xi} \qquad b\le x<\infty \qquad (5.5.4)$$

$$\left.\begin{aligned} K_a &= \left(K_{51}K_8+K_{61}K_7\right)/K_9 \\ K_b &= \left(K_{51}K_6+K_{65}K_7\right)/K_9 \\ K_c &= \left(K_{71}K_8+K_7K_{81}\right)/K_9 \\ K_d &= \left(K_{71}K_6+K_{81}K_5\right)/K_9 \end{aligned}\right\} \quad (5.5.5)$$

$$\left.\begin{aligned} K_{51} &= K_1+K_0\left\{1+2(1-\eta)(1-r_1)\right\} \\ K_{61} &= K_2+2K_0(1-\eta) \\ K_{71} &= K_0\left\{1-(1-r_1)(1-2\eta)\right\}+r_1\left(r_1a_{11}-a_{12}a_{66}\right) \\ K_{81} &= a_{11}+a_{66}-a_{11}r_2+K_0(1-2\eta) \end{aligned}\right\} \quad (5.5.6)$$

Now substituting the value of $\emptyset(\xi)$ and $\psi(\xi)$from

(5.3.7)-(5.3.9) and evaluating the integrals, thus we get.

Iso orth.

$$u_x(x,0)+u_x(x,0)=K_a\left[P_4(b)-\int_b^x g(t)dt\right]$$

$$+K_b\left[\frac{1}{\pi}\int_b^\infty h(t)\log\left|\frac{x+t}{x-t}\right|dt-\int_0^b P_5(t)\log\left|\frac{x+t}{x-t}\right|\right]$$

$-K_0P_4(x)$ $b \le x \le x$ (5.5.7)

Iso orth

$$u_y(x,0)+u_y(x,0)=\frac{K_c}{\pi}\int_b^x g(t)\log\left|x^2-t^2\right|dt$$

$$-\frac{K_c}{\pi}\int_b^x P_4(t)\log\left|x^2-t^2\right|dt-Kd\int_x^\infty h(t)dt$$

$-K_0P_5(x)$ $b \le x \le \infty$ (5.5.8)

Thus we can early evaluate $u_y(x,0^+)$ and $u_y(x,0_-)$ $u_x(x,0^-)$ through relations (5.5.2), (5.5.6) and (5.5.1) and (5.5.7) respectively

**Stress Components**

First we evaluate shear stress in the vicinity of crack tips we know from the continuity condition that

Iso orth

$\sigma_{xy}(x,0) - \sigma_{xy}(x,0),\quad 0 \le x < b$ (5.5.9)

We next find the value of

Iso orth

$$\sigma_{xy}(x,0)+\sigma_{xy}(x,0)=\int_b^\infty \xi[K_{10}\emptyset(\xi)-K_{11}\psi(\xi)],$$

$0 \le x < b$ (5.5.10)

$sin(\xi x)d\xi - P_6(x)$

Substituting the value of $\emptyset(\xi)$ and $\psi(\xi)$ from

(5.3.7)-(5.3.8) and evaluating the integrals, we get as

Iso orth

$$\sigma_{xy}(x,0)+\sigma_{xy}(x,0)=-\frac{K_{10}x}{\pi}\int_b^\infty \frac{g(t)dt}{x^2-t^2}$$

$+\frac{K_{10}x}{\pi}\int_0^b \frac{P_4(t)dt}{x^2-t^2} - P_6(x),\ 0 \le x < b$ (5.5.11)

Iso orth

$$\sigma_{xy}(x,0)+\sigma_{yy}(x,0)=\int_b^\infty \xi[K_{12}\emptyset(\xi)+K_{13}\psi(\xi)],$$

$cos(\xi x)d\xi - \int_0^\infty P_1(\xi)\,cos(\xi x)d\xi,\ 0\le x<b$ (5.5.12)

Now substituting from (5.3.7)-(5.3.8) into (5.5.10) we get

Iso orth

$$\sigma_{yy}(x,0)+\sigma_{yy}(x,0)=\frac{K_{13}}{\pi}\int_b^\infty \frac{h(t)tdt}{x^2-t^2}-$$

$$\frac{K_{13}}{\pi}\int_0^b \frac{P_5(t)tdt}{x^2-t^2}-\int_0^\infty P_1(\xi)cos(\xi x)\,d\xi,$$

$0\ \le x<\ b$ (5.5.13)

Thus substituting the solutions for h(t) and g(t) we can evaluate the physical quantities.

**5.6 SPECIAL CASE OF LOADING**

We assume that the crack opening is due to constant and uniform force at crack faces, see figure 5.3

$P_1(x)=P_2(x)=P_0$ (5.6.1)

Using (5.6.1) in (5.2.9), we get

$P_1(\xi)=0$ (5.6.2)

The value of $P_1(\xi)$ is used in (5.2.16)-(5.2.17) which gives

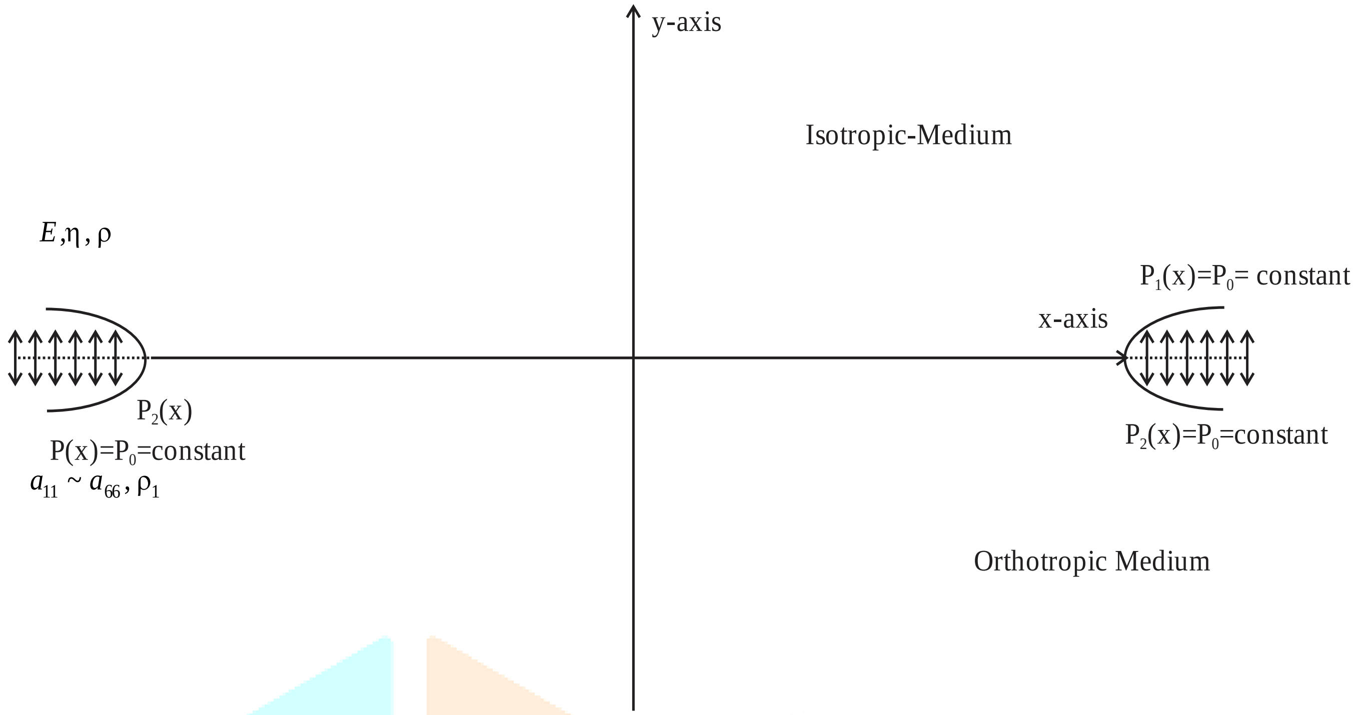


**Fig 5.3 Geometry of problem with Special Loading**

$$P_4(x) = P_5(x) = P_6(x) = 0 \tag{5.6.3}$$

$$P_7(x) = P_0 \tag{5.6.4}$$

Using (5.6.2)-(5.6.4) into (5.3.9), (5.3.12)-(5.3.15) we get

$$P_8(x) = P_0 \tag{5.6.5}$$

$$P_9(x) = 0 \tag{5.6.6}$$

$$P_{10}(x) = -K_{13}h(x) \qquad b < x < \infty \tag{5.6.7}$$

$$P_{11}(x) = P_0 + K_{10}g(x) \quad b < x < \infty \tag{5.6.8}$$

$$P_{12}(t) = \frac{1}{\pi K_{11}\sqrt{t^2 - b^2}}\left[D_2 + D_1\phi(t) + \frac{\sqrt{t^2 - b^2}}{2t}\right.$$

$$\left.\left[\log\frac{t - \sqrt{t^2 - b^2}}{t + \sqrt{t^2 - b^2}}\right]\right] \qquad b < t < \infty \tag{5.6.9}$$

$$I_0(t)=\frac{K_0K_{12}}{\pi^2K_{11}^2}\frac{1}{\sqrt{t^2-b^2}}\left[D_2\left\langle\phi_0^2(t)-\phi_2^1(t)\right\rangle\right]$$

$$+\left[D_1\left\langle\phi_0(t)\phi_2^1(t)-\phi_2^2(t)\right\rangle+\phi_0(t)\phi_3^1(t)-\phi_3^1(t)\right]$$

(5.6.10)

With

$$\phi_2^n(t)=\int_b^\infty\frac{\phi_0^n(\alpha)d\alpha}{\alpha^2-t^2}n=1,2$$

(5.6.11)

$$\phi_3^n(t)=\int_b^\infty\frac{\sqrt{\alpha^2-t^2}}{\alpha\left(\alpha^2-t^2\right)}\phi_0^n(\alpha)\log\left|\frac{\alpha+\sqrt{\alpha^2-b^2}}{\alpha-\sqrt{\alpha^2-b^2}}\right|d\alpha\quad n=0,1$$

(5.6.12)

Thus we can easily evaluate h(t) through (5.4.6)-(5.4.10) and (5.6.9)-(5.6.12) the constants $D_1$ and $D_2$ are to be evaluated through.

$$\int_b^\infty g(t)=0 \qquad (5.6.13)$$

$$\int_b^\infty h(t)=0 \qquad (5.6.14)$$

And the evaluated expressions of h(t) and g(t) Therefore, the expressions of crack opening displacements and of stress components in the viscinity of crack tips can easily be calculated.

**Refrences**

i. Dwivedi A.P A griffth crack opened by

Kushwaha P.S. symmetrical system by body and Awasthi R.D forces in stress free-strip

ISTAM Proc. (1978) PP 15-20

ii. Sneddon, I.N. "The crack intensity factor

for a griffth-crack in an elastic body in which body forces are acting".

Int. J. Fracture mech 3(1967)-P317

iii. Dwrivedi, A.P. N-Barenblatt cracks at interface

and Sharama J.P. of two bonded dissimilar elastic half planes, Acta ciencia Indica, 7 (1981) P.P 25-33

iv. Dwrivedi, A.P. Two Griftth cracks

and Jripathi, P.K. Opened by healed wedge in an infinite Isotropic medium, Acta

ciencia Indica, (2001)

v. Awasthi A.K. "Bone crack inspired

kaur Harpreet, pair of Grifttn

Rachana arel crack opened by

Shavej Ali siddiqui forces at crack faces".

Taylor and francis onhix (2021)

vi. Srivastava, K.N. "Finite Hilbert transform

and Lowengrub, M. technique for triple integral equations with trigonometric kernels'', Proc. Roy. Soc. Edinb. 68.(1968), pp, 309-321.